$$e^1 \wedge e^2 \wedge e^3 \underset{\text{associativity}}{=\!=} (e^1 \wedge e^2) \wedge e^3 \ (= e^1 \wedge (e^2 \wedge e^3))$$

Consider $(e^1 \wedge e^2) \wedge e^3$ and its evaluation

$$(\underbrace{(e^1 \wedge e^2)}_{\in \Lambda^2 V^*} \wedge \underset{\Lambda^1 V^*}{\underset{\cap}{e^3}})(e_1 \otimes e_2 \otimes e_3)$$

$$= \frac{(2+1)!}{2!\,1!} \mathrm{Alt}((e^1 \wedge e^2) \otimes e^3)(e_1 \otimes e_2 \otimes e_3)$$

$$= \frac{1}{2!\,1!} \sum_{\sigma \in S_3} ((e^1 \wedge e^2) \otimes e^3)(e_{\sigma(1)} \otimes e_{\sigma(2)} \otimes e_{\sigma(3)})$$

$$= \frac{1}{2} ((e^1 \wedge e^2) \otimes e^3)(\underset{\sigma = \mathbb{1}}{e_1 \otimes e_2 \otimes e_3} + \underset{\sigma = (12)}{e_2 \otimes e_1 \otimes e_3}$$

Pointed by 李锦锋：this should be a "minus" because in the summation of Alt, there should be a sign "sgn" term appearing.

$+ \cdots$ ← no need to consider other permutation (WHY?))

$$= \frac{1}{2} ((e^1 \wedge e^2)(e_1 \otimes e_2) + (e^1 \wedge e^2)(e_2 \otimes e_1))$$

$$= \frac{1}{2} \left(\frac{(1+1)!}{1!\,1!} \mathrm{Alt}(e^1 \otimes e^2)(e_1 \otimes e_2) + \frac{(1+1)!}{1!\,1!} \mathrm{Alt}(e^1 \otimes e^2)(e_2 \otimes e_1) \right)$$

$$= \frac{1}{2} \left(\sum_{\sigma \in S_2} (e^1 \otimes e^2)(e_{\sigma(1)} \otimes e_{\sigma(2)}) + \sum_{\sigma \in S_2} (e^1 \otimes e^2)(e_{\sigma(2)} \otimes e_{\sigma(1)}) \right)$$

$$= \frac{1}{2} ((e^1 \otimes e^2)(\underset{\sigma = \mathbb{1}}{e_1 \otimes e_2}) + (e^1 \otimes e^2)(\underset{\sigma = (12)}{e_1 \otimes e_2}))$$

This is correct, plus sign, since two minus signs cancel each other.

$$= \frac{1}{2}(1+1) = 1 \quad \checkmark$$